Sydney Girls High School

2003

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics

This is a trial paper ONLY. It does not necessarily reflect the format or the contents of the 2002 HSC Examination Paper in this subject.

Candidate Number

General Instructions

- Reading Time - 5 mins
- Working time - 3 hours.
- Attempt ALL questions
- ALL questions are of equal value
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Standard integrals are supplied
- Board-approved calculators may be used.
- Diagrams are not to scale
- Each question attempted should be started on a new sheet. Write on one side of the paper only.

QUESTION 1

Marks

(a) Factorise $5x^2 - 2x - 3$ **1**

(b) Evaluate correct to one decimal place **2**

$$\frac{3.24}{\sqrt{9.75 - 3.58}}$$

(c) If $h(x) = x^2 - x + 3$ **3**

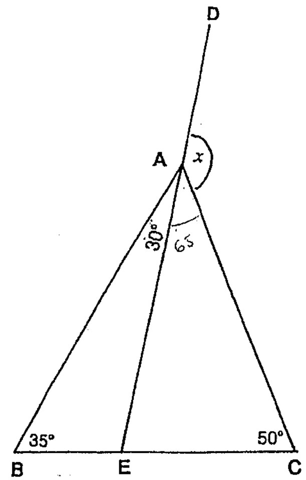
(i) Evaluate $h(-2)$

(ii) For what values of x is $h(x) = 45$

(d) Express $0.5\dot{8}$ as a fraction. **2**

(e) Solve $|x - 5| = 6$ **2**

(f) Find the value of x **1**



NOT TO SCALE

(g) Solve $2 - 3p < 7$ **1**

QUESTION 2

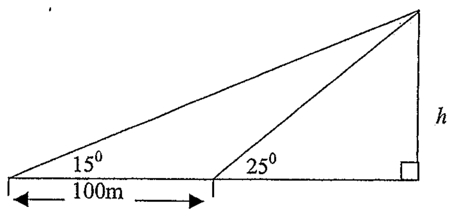
Marks

(a) Differentiate: 4

(i) $\frac{1}{2\sqrt{x}}$

(ii) $x^2 e^x$

(b) A girls finds the angle of elevation of the top of a cliff to be 15^0. She then walks 100m directly towards the base of the cliff and finds the angle of elevation of the top to now be 25^0. How high is the cliff? 3



(c) Find: 5

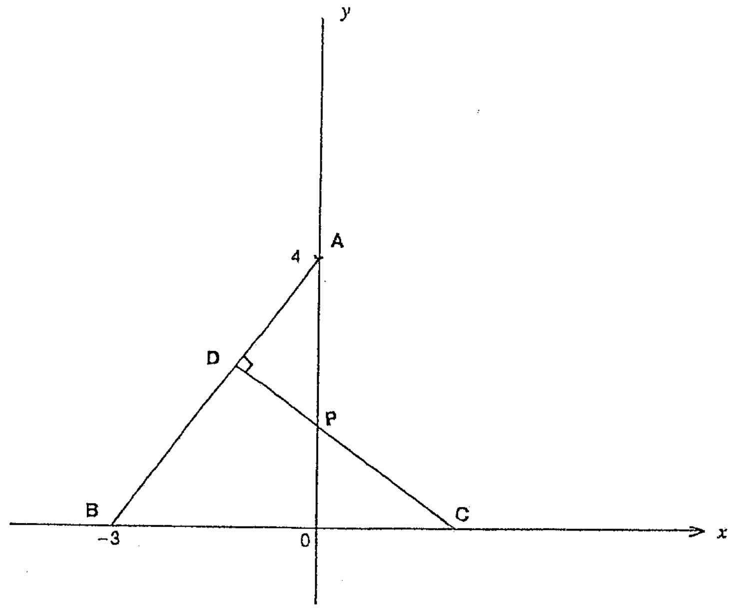
(i) $\int \sec^2 3x\, dx$

(ii) $\int (5x-3)^5\, dx$

(iii) $\int_{-1}^{0} \frac{dx}{2x+3}$

QUESTION 3

Marks



In the diagram AB = BC and CD is perpendicular to AB.
CD intersects the y axis at P.

Copy the diagram onto your answer sheet.

(a) Find the length of AB. 1

(b) Hence show the co-ordinates of C are (2,0) 1

(c) Show the equation of CD is $3x + 4y = 6$ 3

(d) Show the coordinates of P are (0, 1½) 1

(e) Use Pythagoras' Theorem on Δ POC to show the length of CP is 2½ units 1

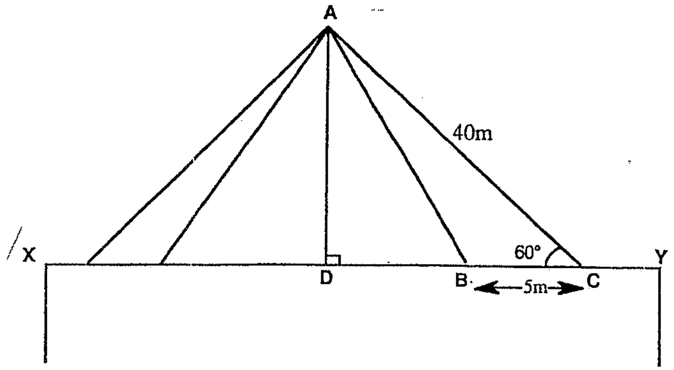
(f) Prove that Δ ADP is congruent to Δ COP. 3

(g) Hence calculate the area of the quadrilateral DPOB. 2

QUESTION 4

Marks

(a) 4



NOT TO SCALE

A horizontal bridge was built between points X and Y.
Cables were used to support the bridge as shown in the diagram above.
The distance between the cables AB and AC was 5 metres.
Cable AC was 40 metres long and $\angle ACB = 60^0$.

(i) Show that the height of A above the horizontal bridge is $20\sqrt{3}$ metres.

(ii) Use the cosine rule to show the exact length of the cable AB is $5\sqrt{57}$ metres.

(b) An arithmetic series has a sum given by $S_n = 25n - 2n^2$. 6

(i) Find the second term (i.e. $S_2 - S_1$)

(ii) Show that the n th term is $T_n = 27 - 4n$

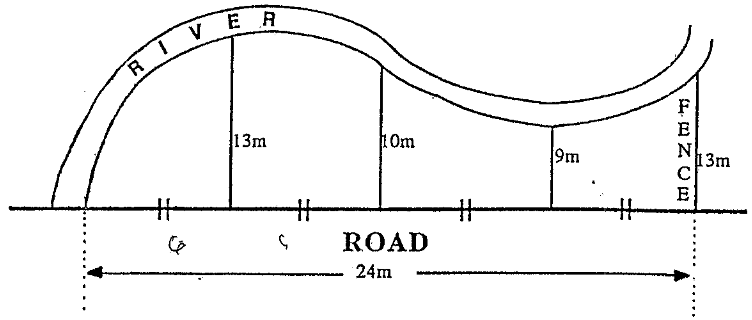
(iii) Find the first term of this series greater than -500

(c) One case contains 6 biros and 5 pencils and a second case contains 8 pencils and 6 biros. If a case is selected at random and one piece of writing equipment chosen at random from it, find the probability that a biro will be chosen. 2

QUESTION 5

Marks

(a) 4



NOT TO SCALE

Wasteland bordering a river bank and a straight road was fenced off and used as a recreational park. Perpendicular distances from the road to the river bank are shown on the diagram.

Use Simpson's Rule, with 5 function values, to approximate the area of the recreational park.

(b) A parabola has equation: 3

$$x^2 - 12y + 2x + 1 = 0$$

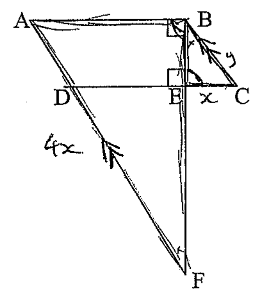
(i) Find the vertex of this parabola.

(ii) What are the co-ordinates of the focus?

(iii) Name the equation of the directrix.

(c) In the diagram: 5

$FB \perp AB$, $EC \perp EB$ and $AF \parallel BC$.



(i) Prove that $\Delta CBE \parallel\!\mid \Delta ABF$

(ii) If $EC = x$ units, $BC = y$ units and AF is four times the length of EC, find the length of AB in terms of x and y.

QUESTION 6.

Marks

(a) Find the equation of the tangent to the curve $y = x \ln x$ at the point (1,0) **3**

(b) Given $\dfrac{\sqrt{5}}{\sqrt{5}-2} = a + \sqrt{b}$ Find a and b **2**

(c) Consider the curve $y = x^3 - 3x$. **7**

(i) Name the three points where the curve cuts the x axis.

(ii) Find the stationary points and determine their nature.

(iii) Find the point of inflexion.

(iv) Sketch the curve showing all relevant features.

QUESTION 7

(a) Mary joins a firm and pays \$500 at the beginning of each year into a superannuation fund, and received interest compounded at 9 per cent per annum. She leaves the firm after 15 years. How much superannuation has she accrued? **4**

(b) For what values does k does the equation **4**

$x^2 - (k + 1)x = -1$ *have:*

(i) equal roots

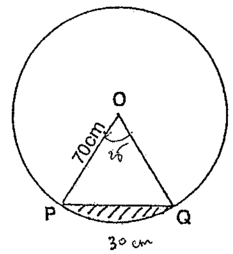
(ii) no real solution

(c) Find the equation of the line through the point of intersection of $5x - 2y + 3 = 0$ and $2x + 6y + 1 = 0$ and also passing through the point (1, –2). **4**

Marks

QUESTION 8

(a) **3**



NOT TO SCALE

In the diagram the length of the arc PQ is 30 cm.
The radius of the circle is 70 cm.

(i) Show that $\angle POQ = 25^0$ to the nearest degree.

(ii) Find the shaded area correct to the nearest cm^2.

(b) The region bounded by the curve $y = x^3$, the y-axis and the line $y = 8$ is rotated about the y axis. Find the volume of the solid formed. **3**

(c) Find the area bounded by the curves $y = e^x$ and $y = e^{-x}$ and the line $x = 2$. **3**

(d) If $2^{2x+1} \cdot 4^{3x} = 8^{x-1}$, find the value of x. **3**

QUESTION 9.

Marks

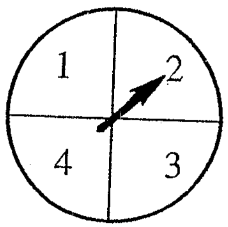
(a) If $3x^2 + 4x + 5 \equiv A(x+1)^2 + B(x+1) + C$
Find the constants A, B and C — **3**

(b) If α and β are the roots of the equation — **3**

$$3x^2 - 2x + 1 = 0$$

evaluate
(i) $\alpha + \beta$
(ii) $\alpha\beta$
(iii) $\dfrac{1}{\alpha^2\beta} + \dfrac{1}{\beta^2\alpha}$

(c) — **3**



Peter and Chris used the spinner shown above to play a game. Peter spun the spinner twice and added the results of the two spins to get his score. Chris then took his turn. The player with the highest score won the game.

(i) Use a diagram to show all the possible scores Peter could have achieved when he played the game.

(ii) What is the probability that Peter scored 6 in the game?

(iii) Peter's score was 6. What is the probability that Chris won the game?

(d) (i) Sketch the graph of $y = \sin x + \cos x$, for $0 \le x \le 2\pi$ — **3**

(ii) Use your sketch to solve the equation $\sin x + \cos x = 0$ in the domain $0 \le x \le 2\pi$.

QUESTION 10.

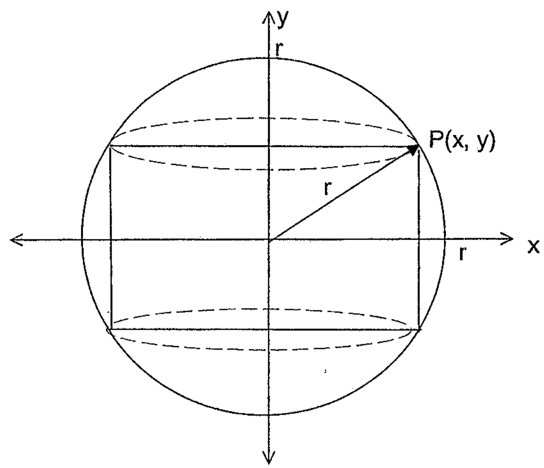
Marks

(a) Find the equation of the normal to the curve — ~~3~~ 2
$y = \sin 2x$ at the point $\left(\dfrac{\pi}{2}, 0\right)$
(Leave your answer in terms of π.)

(b) Given the series $1 + \dfrac{2x}{1+x^2} + \dfrac{(2x)^2}{(1+x^2)^2} + \ldots$ — ~~3~~ 4
for what values of x does this series have a limiting sum?

(c) A sphere, centre corresponding to the intersection of the x and y axes has radius r units (see diagram). — 6

A cylinder is to be cut from the sphere as shown in the diagram.
The point $P(x, y)$ lies on the circle $x^2 + y^2 = r^2$.



(i) Show the volume of the cylinder is $V = 2\pi(r^2 - y^2).y$ — **3**

(ii) Find the maximum volume of this cylinder in terms of r. — **4**

End of Paper

SGHS 2u Trials '03

Question 1

(a) $5x^2-2x-3=(5x+3)(x-1)$ ✓

(b) $\dfrac{3.24}{\sqrt{6.17}} = 1.3$ (to 1 dp) ✓

(c) (i) $h(-2) = 4+2+3 = 9$ ✓

(ii) $45 = x^2 - x + 3$

$\therefore x^2 - x - 42 = 0$

$(x-7)(x+6) = 0$ ✓

$\therefore x = 7, -6$ ✓

(d) $0.58 +$
$0.008 +$
0.0008

$\therefore a = 0.008$

$r = \frac{1}{10}$ $\therefore S_\infty = \dfrac{0.008}{0.9} = \dfrac{2}{225}$ ✓

$\therefore 0.58 + \dfrac{12}{225} = \dfrac{53}{90}$ ✓

(e) $x-5=6$ or $x-5=-6$

$x = 11$ ✓ or $x=-1$ ✓

(f) $A\hat{E}C = 65°$ ✓ (ext ∠ of Δ = sum of opp int ∠s)

$\therefore x = 65+50$ (" ")

$= 115°$ ✓

(g) $-3p < 5 \Rightarrow p > -\frac{5}{3}$.

Question 2

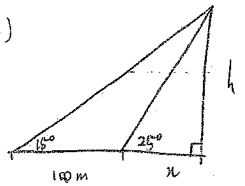
(a) (i) $\dfrac{d}{dx}\ \frac{1}{2}x^{-\frac{1}{2}} = -\frac{1}{4}x^{-\frac{3}{2}}$

$= -\dfrac{1}{4\sqrt{x^3}}$ ✓

(ii) $\dfrac{d}{dx}\ x^2e^x = 2xe^x + x^2e^x$ ✓

$= xe^x(2+x)$

(b)



$\tan 15 = \dfrac{h}{100+x}$ — ① $\tan 25 = \dfrac{h}{x}$ — ②

$100\tan 15 + x\tan 15 = h$ $x\tan 25 = h$

$\dfrac{h - 100\tan 15}{\tan 15} = x$ $x = \dfrac{h}{\tan 25}$ ✓

$\therefore$ equating equs: ✓

$\dfrac{h-100\tan 15}{\tan 15} = \dfrac{h}{\tan 25}$ ✓

$h\tan 25 - 100\tan 15\tan 25 = h\tan 15$

$\therefore h(\tan 15 - \tan 25) = -100\tan 15\tan 25$

$\therefore h = 62.99$m (to 2dp) ✓

(c) (i) $\int \sec^2 3x\,dx = \frac{1}{3}\tan 3x + C$ ✓

(ii) $\int (5x-3)^5\,dx = \frac{1}{5}\dfrac{(5x-3)^6}{6} + C$

$= \dfrac{(5x-3)^6}{30} + c$

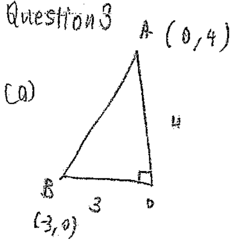
(iii) $\int_{-1}^{0} \dfrac{1}{2x+3}\,dx = \left[\frac{1}{2}\ln(2x+3)\right]_{-1}^{0}$

$= \frac{1}{2}\ln 3 - \frac{1}{2}\ln 1$ ✓

$= \frac{1}{2}\ln 3$ ✓

Question 3

(a)



$\therefore AB = 5$ (pythagorean triad) ✓

(b) $AB = BC$

$\therefore$ BC is 5 units long

$\therefore$ C must be $(2,0)$ ✓

to make $BC = \sqrt{25} = 5$ units

(c) C $(2,0)$ ✓

$m_{AB} = \dfrac{4}{3}$

$\therefore m_{CD} = \dfrac{-3}{4}$

$\therefore y = \frac{-3}{4}(x-2)$ ✓

$4y = -3x+6$

$\therefore 3x+4y=6$ is eqn of CD

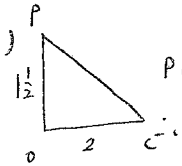
(d) when CD crosses y axis, $x=0$

$\therefore 4y = 6$

$y = \frac{3}{2}$

$\therefore P = (0, 1\frac{1}{2})$ ✓

(e)



$PC^2 = (1\frac{1}{2})^2 + 4$

$\therefore CP = \sqrt{\frac{25}{4}}$

$= \frac{5}{2}$ units ✓

(f) In ΔADP, ΔCOP

$A\hat{P}D = O\hat{P}C$ (vert opp ∠s =) ✓

$\angle ADP = \angle POC = 90°$ ✓ (∠ b/t axis is 90°)

$AP = OP$ (given) ✓

$\therefore \Delta ADP \equiv \Delta COP$ (AAS)

(g) A of Δ BOP $= \frac{1}{2} \times 3 \times 1\frac{1}{2}$

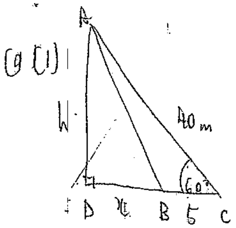
$= 2.25$ units² ✓

A of DPOB $= 2 \times 2\frac{1}{4}$ ✓

$= 4\frac{1}{2}$ units²

Question 4

(a) (i)



$\sin 60 = \frac{h}{40}$

$\frac{\sqrt{3}}{2} = \frac{h}{40}$ ✓

$\therefore h = 20\sqrt{3}$ m ✓

(ii) $AB^2 = 40^2 + 5^2 - 2(40)(5)\cos 60$

$= 1425$

$AB = \sqrt{1425}$ ✓

$= 5\sqrt{57}$ m ✓

(b) (i) $S_1 = 25 - 2 = 23$ ✓

$S_2 = 50 - 2(4)$

$= 42$ ✓

$\therefore T_2 = S_2 - S_1$

$= 19$ ✓

(ii) $a = 23$

$d = -4$ ✓

$\therefore T_n = 23 + (n-1)(-4)$

$= 27 - 4n$ ✓

(iii) $T_n > -500$

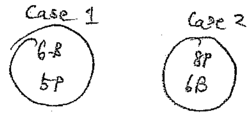
$\therefore 27 - 4n > -500$

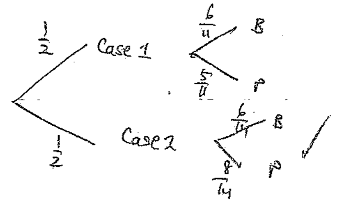
$4n < 527$ ✓

$n < \frac{527}{4}$

$n = 131$ ✓

(c)





$\therefore P(B) = \frac{1}{2} \times \frac{6}{11} + \frac{1}{2} \times \frac{6}{14}$

$= \frac{75}{154}$ ✓

Question 5

(a)

x	0	6	12	18	24
$f(x)$	0	13	10	9	13

✓

$A = \int_0^{24} f(x) \doteq \frac{12}{6}\left[13 + 4(22) + 20\right]$

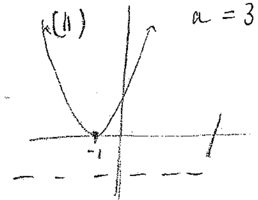
$\doteq 242$ m² ✓

(b) (i) $x^2 + 2x + 1 = 12y$

$(x+1)^2 = 12y$

$\therefore V(-1, 0)$ ✓

(ii) $a = 3$ $\therefore S = (-1, 3)$ ✓



(iii) $y = -3$ ✓

(c) (i) In ΔCBE, ΔABF

$\angle BEC = 90°$ (straight ∠) ✓

$= \angle ABF$ (given)

$\angle CBE = \angle AFB$ (alt ∠ of ∥ lines AF, BC)

$\therefore \Delta CBE \parallel\!\mid \Delta ABF$ (equiangular)

(ii) $\frac{CE}{AB} = \frac{BC}{AF}$

let $CE = x$, $BC = y$ ∴ $AF = 4x$

$\therefore \frac{x}{AB} = \frac{y}{4x}$

$\therefore AB = \frac{4x^2}{y}$ units ✓

Question 6

(a) $y = x\ln x$

$y' = \ln x + 1$ ✓

when $x = 1$, $m = 1$ ✓

$\therefore y - 0 = x - 1$

$y = x - 1$ is equ of tangent ✓

(b) $\frac{\sqrt{5}(\sqrt{5}+2)}{5-4} = 5 + 2\sqrt{5} = 5 + \sqrt{20}$ ✓

✓ $\therefore a = 5, b = 20$ ✓

(c) $y = x^3 - 3x = x(x^2 - 3)$ ✓

(i) cut x axis when $y = 0$

$\therefore (0,0), (\sqrt{3}, 0), (-\sqrt{3}, 0)$ ✓

(ii) $y' = 3x^2 - 3$

when $y' = 0$, $3(x^2 - 1) = 0$

$x = 1, -1$

$y'' = 6x$

$\therefore$ at $x = 1$, $y = -2$

$y'' > 0$ ✓

$\therefore$ min turning pt at $(1, -2)$

at $x = -1$, $y = 2$

$y'' < 0$ ✓

$\therefore$ max turning pt at $(-1, 2)$

(iii). when $y'' = 0$

$6x = 0$

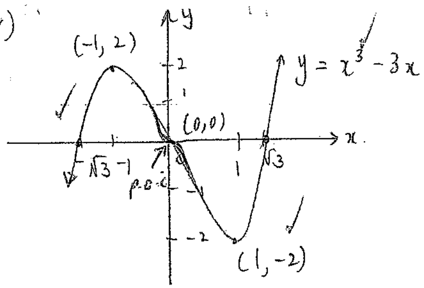
$x = 0$ ∴ possible poi at $x = 0$

x	-1	0	1
y''	$+$	0	$-$

∴ since Δ in concavity,

poi at $(0, 0)$

(iv)



(b)(i) equal roots when $\Delta = 0$

$x^2 - (k+1)x + 1 = 0$

$\Delta = b^2 - 4ac$

$= k^2 + 2k + 1 - 4$

$= k^2 + 2k - 3$

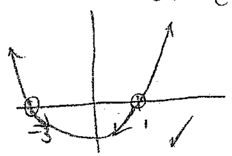
∴ $(k+3)(k-1) = 0$

$k = -3, 1$

(ii) no real soln when $\Delta < 0$

∴ $(k+3)(k-1) < 0$

∴ $-3 < k < 1$



Qu 7 (a) $M = 500$

$R = 1.09$

$n = 15$

Let $A_1 = 500(1.09)^{15}$

$A_2 = 500(1.09)^{14}$

⋮

$A_{15} = 500 \times 1.09$

∴ $S_{1-15} = \dfrac{500(1.09)(1.09^{15} - 1)}{0.09}$

$= \$16\,001.70$

∴ accrued \$16001.70

(c) let eqn of line thru pt of intersection be in form

$(a_1x + b_1y + c_1) + k(a_2x + b_2y + c_2) = 0$

∴ $(5x - 2y + 3) + k(2x + 6y + 1) = 0$

sub in $(1, -2)$

$5 + 4 + 3 + k(2 - 12 + 1) = 0$

$12 + -9k = 0$

∴ $k = \frac{4}{3}$

∴ $3(5x - 2y + 3) + 4(2x + 6y + 1) = 0$

$15x - 6y + 9 + 8x + 24y + 4 = 0$

$23x + 18y + 13 = 0$

Question 8

(a) $l = r\theta$

$30 = 70\theta$

$\theta = \frac{3}{7}$ rad

$1 \text{ rad} = \frac{180}{\pi}$

∴ $\theta = \frac{3}{7} \times \frac{180}{\pi} = 24°33'$

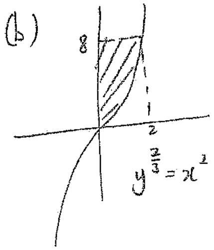
$= 25°$ (nearest deg)

(ii) shaded $A = \frac{1}{2}r^2(\theta - \sin\theta)$

$= 31.85 \text{ cm}^2$

$= 32 \text{ cm}^2$ (nearest cm²)

(b)



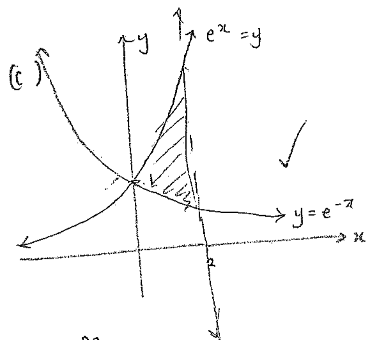
$V = \pi \int x^2 \, dy$

$= \pi \int_0^8 y^{\frac{2}{3}} \, dy$

$= \pi \left[\frac{3}{5} y^{\frac{5}{3}} \right]_0^8$

$= 19\frac{1}{5}\pi$ units³

(c)



$A = \int_0^2 e^x - e^{-x} \, dx = \left[e^x + e^{-x} \right]_0^2$

$= e^2 + e^{-2} - (e^0 + e^0)$

$= \left(e^2 + \frac{1}{e^2} - 2\right)$ units²

(d) $2^{2x+1} \cdot 2^{6x} = 2^{3x-3}$

$2^{8x+1} = 2^{3x-3}$

∴ $8x + 1 = 3x - 3$

$5x = -4$

$x = -\frac{4}{5}$

Question 9

(a) $3x^2 + 4x + 5 \equiv A(x+1)^2 + B(x+1) + C$

let $x = -1$,

$3 - 4 + 5 = C$

∴ $C = 4$

let $x = 0$,

$5 = A + B + C$

$A + B = 1$

$A = 3$ $(Ax^2 = 3x^2)$

∴ $B = -2$

∴ $A = 3$

$B = -2$

$C = 4$

(b) (i) $\alpha + \beta = \frac{-b}{a} = \frac{2}{3}$ ✓

(ii) $\alpha\beta = \frac{c}{a} = \frac{1}{3}$ ✓

(iii) $\frac{\alpha+\beta}{\alpha^2\beta^2} = \frac{\frac{2}{3}}{\frac{1}{9}}$

$= 6$ ✓

(c) (i)

	1	2	3	4
1	2	3	4	5
2	3	4	5	6
3	4	5	6	7
4	5	6	7	8

✓

(ii) $P(6) = \frac{3}{16}$ ✓

(iii) $P(\text{Chris win}) = \frac{3}{16}$ ✓

(d) (i) $y = \sin x + \cos x$ for $0 \le x \le 2\pi$

let $y = \sin x + \cos x$ be in form $R\sin(x + \alpha)$

$R(\sin x \cos\alpha + \cos x \sin\alpha)$

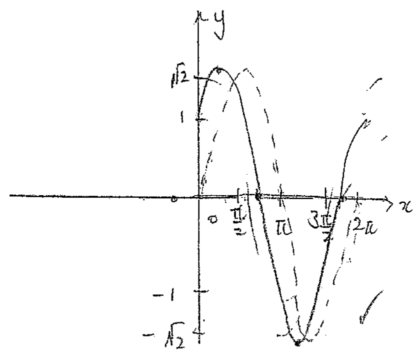
$R\cos\alpha = 1$

$R\sin\alpha = 1$

$\therefore R = \sqrt{2}$

$\tan\alpha = 1$

$\alpha = \frac{\pi}{4}$



(ii) $\sin x + \cos x = 0$

$x = \frac{3\pi}{4}, \frac{7\pi}{4}$ ✓

Question 10

(a) $y' = 2\cos 2x$

when $x = \frac{\pi}{2}$, $m = -2$

$m_{\perp} = \frac{1}{2}$ ✓

$y = \frac{1}{2}(x - \frac{\pi}{2})$ ✓

$= \frac{1}{2}x - \frac{\pi}{4}$

(b) $r = \frac{2x}{1+x^2}$

So when $-1 < r < 1$

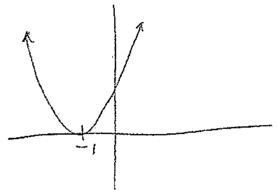
$\therefore -1 - x^2 < 2x < 1 + x^2$

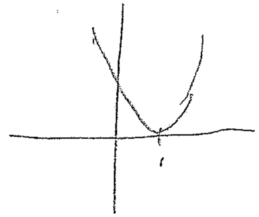
$-1 - x^2 < 2x$ or $2x < 1 + x^2$

$x^2 + 2x + 1 > 0$ ✓ or $x^2 - 2x + 1 > 0$

$(x+1)^2 > 0$ $(x-1)^2 > 0$ ✓

∴ True for all x.





∴ To have limiting sum, $x \neq \pm 1$ ✓

(c) (i) $V = \pi R^2 h$

$= \pi x^2 2y$ $x^2 = r^2 - y^2$ ✓

$= \pi(r^2 - y^2)2y$

$= 2\pi(r^2 - y^2)y$ ✓

(ii) $V = 2\pi y r^2 - \pi y^3$

$\frac{dV}{dy} = 2\pi y r - 3\pi y^2$

$\pi r^2 - 3\pi y^2 = 0 \Rightarrow y^2 = \frac{\pi r^2}{3\pi}$

$= \frac{r^2}{3}$

$y = \pm\frac{r}{\sqrt{3}}$

when $\frac{dV}{dy} = 0$, $\pi y(2r - 3y)$ ✗

$y = 0$ or $y = \frac{2r}{3}$

$\frac{d^2V}{dy^2} = 2\pi r - 6\pi y$ when $y = \frac{2r}{3}$ ✗

$\frac{d^2V}{dy^2} = 2\pi r - 4\pi r$

$= -2\pi r$ ✗

2r

$\therefore V = 2\pi(r^2 - \frac{4r^2}{9})(\frac{2r}{3})$

$= \frac{4\pi r^3}{3} - \frac{8\pi r^3}{27}$

$= \frac{28\pi r^3}{27}$ units

$\therefore V = 2\pi(r^2 - \frac{r^2}{3})\frac{r}{\sqrt{3}}$

$= 2\pi(\frac{2r^2}{3})\frac{r}{\sqrt{3}}$

$= \frac{4\pi r^3}{3\sqrt{3}}$

$= \frac{4\sqrt{3}\pi r^3}{9}$ u^3